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Fire Fragility Functions for Steel Frame Buildings: Sensitivity Analysis and Reliability Framework

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Abstract. Fire fragility functions are a powerful method to characterize the probabilistic vulnerability of buildings to fire in the context of urban resilience assessment. But this method is recent and the influence of the different uncertain parameters on the functions has not been systematically studied. The first objective of this paper is to identify the prevailing parameters in constructing fire fragility functions for steel frame buildings. To this end, sensitivity analyses are conducted using Monte Carlo Simulations and a variance-based method, focusing on column failure fragilities. Fragilities for buildings with 3 to 12 stories, 0 to 3 h fire resistance rating and various occupancies are compared, assuming compartment areas ranging from 15 m^2 to 80 m^2. Results show that uncertainties in fire, heat transfer and structural models all generate significant variability in the fire fragility. In addition to fire load as the intensity measure, significant probabilistic parameters are the compartment geometry and openings, the thickness and thermal conductivity of fire protection, and the temperature dependent mechanical properties of steel. The second objective is to clarify the incorporation of fragility functions in a comprehensive structural fire reliability framework. A methodology for combining the functions with the ignition likelihood per year and with the fire loading in MJ/m^2 is described, yielding annual probability estimates of column failure due to fire in the buildings. For a sprinklered office building designed according to prescriptive provisions, this annual probability ranges from 1.90×10^{-7} to 0.12×10^{-7} per year as a function of the building height. The probabilistic modeling techniques proposed in this paper can be used to establish consistent reliability levels in different buildings and to evaluate resilience for fire scenarios.

Keywords: Fire fragility curve, Structural reliability, Sensitivity analysis, Steel building, Urban resilience

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1. Introduction

The standard approach in fire design of structures is based on design at the element level (e.g., beams, columns) using prescriptive approaches, where uncertainties in variables are not explicitly incorporated in the process. However, data indicate large uncertainty in the values of the parameters affecting the fire behavior of structures, such as fire load [1], thermal conductivity of insulation material [2, 3] or, to some extent, mechanical properties of materials at elevated temperatures [4, 5]. To allow for a successful shift from prescriptive- to performance-based paradigms in building codes relative to fire safety, performance criteria need to be established that are "verifiable and enforceable" [6]. Acknowledging the many uncertainties at stake, these performance criteria are best expressed in a probabilistic framework and linked to safety targets. Hopkin et al. [7] have shown that Probabilistic Risk Analyses (PRA) are necessary to support explicit safety verification, particularly for uncommon fire safety designs for which the collective experience of the profession is insufficient to support an implicitly defined safety level. An efficient and appealing way to capture the effects of uncertainties on the structural fire response of the built environment, in view of performing an explicit safety assessment, is to develop fragility functions.

Fragility functions provide the probability of exceeding a damage state (e.g. column failure, excessive beam deflection, connection failure, etc.) for a given intensity measure of the hazard (fire in this case). The damage states are generally related to the structural performance level and can be grouped in different categories such as 'no damage', 'slight', 'moderate', 'extensive', and 'complete'. Fragility functions can be used for evaluating losses at the scale of a community in the context of disaster resilience assessment [8].

The seismic engineering community largely adopted the approach of using fragility functions. This community developed a suite of seismic fragility functions for different structural typologies (e.g. [9, 10]). The method generally consists of deriving analytical fragility functions based on stochastic analyses of prototype buildings that are assumed representative of a typology. The parameters in the analyses are taken as random variables and Monte Carlo Simulations (MCS) are used to generate the distributions. Alternatively, empirical functions can be developed when sufficient historical damage data is available [11].

In this context, adoption of fragility functions for fire hazard appears as a promising technique. Recently, research in fire engineering is moving towards a performance-based approach that explicitly accounts for uncertainties [12–16]. The literature describes methods for probabilistic analysis of steel [17–19], concrete [20–24], and composite steel beam with concrete slab [25] structural members under fire. However, these methods mainly address the fire reliability of isolated structural members rather than complete structures. Lange et al. [26] have established a methodology for performance-based fire engineering of structures based on the seismic engineering framework developed in the Pacific Earthquake Engineering Research (PEER) Center. Yet additional efforts are needed to develop a methodology that incorporates the uncertainties in fire occurrence, fire development, heat transfer and structural response at the building scale; adoption of fragility functions at a system level constitutes a promising approach.

Gernay et al. [27] recently proposed a novel methodology to generate fire fragility functions providing a probabilistic measure of performance for an entire building system. The fragility functions can be used to evaluate a city's resilience to fire hazard, including in case of multi-hazard cascading event such as fire following earthquake [28, 29]. Rush and Lange [30] used fragility functions for concrete columns in fire, while Marasco et al. [31] used fragility functions in a multi-hazard analysis involving fire. However, the adoption of fire fragility functions is very recent and still requires further research. The methodology proposed in [27] focused on describing the required steps to derive a fire fragility function assuming that the building experiences a structurally significant fire. Yet, essential questions arise in the process of constructing fire fragility functions with respect to the influence of model parameters on fragility functions at the building level and the results interpretation when fragility functions are extended to include probability of having a fire event inside a building. This paper focuses on the following two important questions that are yet unresolved, with the goal to advance the understanding and application of probabilistic approach to structural fire engineering.

The first issue is the influence of the different parameters with uncertainty on the fire fragility functions. A few previous works have presented sensitivity analyses on the fire response of structural members, for instance on simply supported steel beams using a deflection limiting criteria [32]. While these works provide valuable insights, further studies are needed to identify the prevailing parameters that have to be considered as random variables at each step when constructing system fragility functions. Generally, the performance of a building under fire is highly non-linear, so that its evaluation entails the use of advanced computational modeling techniques [33]. The computational time for thousands of simulations required by the application of brute force Monte Carlo Simulation (MCS) techniques hinders the adoption of probabilistic approaches. A large number of input parameters with uncertainty adds to the complexity of analysis and the computational time. Besides, probability distributions are needed for these parameters but rigorous data are often lacking. In order to prioritize the efforts in data collection and limit the complexity of the analyses, it is crucial to identify the parameters that most affect the global fire safety. An in-depth understanding of the sensitivity of the fragility functions to different input parameters, their modeling approaches, and assumptions when aggregated at the system level is necessary for advancing the use of probability approach in the field. Addressing these issues, this paper conducts sensitivity analyses and provides a quantitative comparison of different designs through application of fire fragilities for entire buildings. Notably, the paper provides a cross comparison of fragilities for buildings of different heights and fire resistance rating.

The second issue is to integrate fragility functions within a comprehensive structural fire reliability framework. Specifically, it is necessary to incorporate, in a unique framework, the uncertainties in fire occurrence and location, fire development, heat transfer processes and structural response, at the building scale. This is necessary to systematically obtain the overall probabilities to reach different levels of damage (e.g. slight vs. collapse) for buildings of various typologies, structural design, size and occupancy. Therefore, the second objective of this paper is to propose a methodology for incorporating the fragility functions into a broader framework to assess the risk of structural failure due to fire for multi-story buildings (i.e. incorporation in a PRA). Application on a building type of various heights, occupancies and fire resistance rating is presented for illustration and allows discussing the global reliability levels reached with a prescriptive approach. Once the process of constructing fire fragility functions is well established, the functions can also be used in evaluating and ensuring consistent reliability levels in design of new buildings according to performance-based fire engineering.

2. Method

2.1. Fire Fragility Functions

The overarching goal is to assess the vulnerability of a community to fire disasters. A community comprises a great number of buildings that can be grouped in distinct typologies for the sake of the analysis (for adjusting to the spatial scale of the community). The elements of a typology share attributes and structural features leading them to exhibit a similar response to fire. Therefore, for each typology, fragility functions can characterize this response. Typology definition depends on structural material, structural system, etc.

The methodology for developing fragility functions to quantify vulnerability of a building subject to fire proposed by Gernay et al. [27] is used in this work; it is briefly summarized here. It requires the probabilistic assessment of the structural system performance under fire. The intensity measure selected as the control parameter to characterize the hazard is the fire load. The probabilistic performance assessment takes into account uncertainties in the fire model, the heat transfer model and the structural response, in addition to fire scenarios at different locations in the building. In a multi-story building, fire usually starts and develops locally in a compartment. Then, it may burn out or spread to adjacent compartments. Consequently, fragility functions are derived first for a fire in a well-defined compartment; the latter are referred to as local fragility function FF_L. The process is repeated for each compartment of the building. The local functions are generally different for each fire location within a same building since design parameters vary between compartments. Then, the building fragility function (FF_B) is obtained by combining the local fragility functions (FF_L) corresponding to each fire location. This combination takes into account the conditional probability associated with each FF_L, i.e. the probability to have the fire in the corresponding compartment should a fire occur in the building, by weighting the importance of the FF_L in the global function FF_B. The FF_B characterize the overall vulnerability of the building regardless of the fire location.

At the compartment level (FF_L), the probability of reaching a damage state $P_{F|H_{fi}}(q_i)$, where H_{fi} denotes the fire hazard, must be evaluated for a number of given fire loads q_i (in MJ/m^2). In all generality, this evaluation can be done through complete non-linear analysis of the structure (e.g. using the finite element method). Such advanced calculation methods are required for capturing complex structural response, or with concrete members exhibiting significant thermal gradients that govern the behavior. For some simple situations involving steel members, the evaluation can be done by comparing the demand and capacity of the member. This is the case when temperature distribution can reasonably be considered as uniform in the section and fire-induced forces do not play a significant role. In these cases, MCS are used to generate the probability density function (PDF) of demand and capacity relative to a given damage state. The random variable representing demand is taken as the maximum temperature in the steel section (for a given fire load), T_{max}, whereas capacity is taken as the critical temperature in the steel section relative to the given damage state (i.e. temperature at failure), $T_{critical}$. Convolution of the complementary cumulative distribution function (CDF) of demand $F_D(T, q_i)$ with the PDF of capacity $f_C(T)$ yields the probability of reaching the damage state $P_{F|H_{fi}}(q_i)$. This probability is conditional to the occurrence of a fire H_{fi} and relative to the fire load q_i. It is given by Eq. 1 in which T is the temperature.

$$P_{F|H_{fi}}(q_i) = \int_0^{\infty} [1 - F_D(T, q_i)] f_C(T) dT \tag{1}$$

The computation is performed for several levels of fire load q_i in order to get a number of fragility points $P_{F|H_{fi}}(q_i)$. Then, a fragility function FF_L (q) is fitted to these points, typically assuming a two-parameter lognormal distribution function according to Eq. 2. The lognormal assumption is commonly adopted in earthquake engineering and, although the basis is less clear in fire safety engineering due to a lack of studies, recent results tend to confirm the validity of this assumption for fire [34].

$$FF_L(q) \equiv P_{F|H_{fi}}(q) = \Phi\left[\frac{ln(q/c)}{\zeta}\right] \tag{2}$$

In Eq. 2, $\Phi[\cdot]$ is the standardized normal distribution function. The two parameters c and ζ characterize the fragility function and are determined by maximizing the best fit with the data points resulting from the analysis. The fragility function FF_L (q) yields the probability of reaching the damage state conditional to the occurrence of a fire H_{fi} as a function of the fire load q. In building these functions, the PDFs of demand and capacity of the members are key and therefore the propagation of uncertainty from the parameters influencing those PDFs should be carefully considered.

At the building level (FF_B), the functions have the same mathematical expression as given by Eq. 2, but in which the parameters c and ζ are "weighted combinations" of the parameters of the FF_L. Note that it is an approximation to assume a lognormal distribution for the building (combined) functions but, in line with [11], this assumption is regarded as reasonable when the fragility functions that are combined pertain to structures designed using a same design code. At the building level, design parameters at the building scale influence the fire fragility functions. Therefore, sensitivity of the FF_B to these parameters will be analyzed.

2.2. Sensitivity Analysis

The process of establishing fragility functions involves several steps. At each step, parameters with uncertainty are involved.

The first step consists in evaluating the probability distributions for demand $F_D(T, q_i)$ (maximum temperature reached in the steel) and capacity $f_C(T)$ (critical temperature at failure) for the considered damage state. This evaluation relies on the probabilistic assessment of the fire development and the thermal (heat transfer) response for demand; and the structural response for capacity. These assessments involve a large number of parameters with uncertainty, such as the areas of openings in a compartment, thermal conductivity of an insulation material, and applied gravity loading, among others. Sensitivity analyses are conducted to determine the sensitivity of demand and capacity to the different parameters. The analyses are conducted using Monte Carlo Simulations and a one-at-a-time (OAT) local variance-based method [35]. The demand and capacity distributions are used to calculate the fragility points (Eq. 1) which are used to construct the local fragility function (Eq. 2). Therefore, the sensitivity analyses for demand and capacity provide an insight into uncertainty propagation from the fire model, heat transfer model and structural model to the local fragility functions. The identified key parameters that influence the local fragility functions are compared with previously reported results in the literature. This way, effect of different modeling approaches in defining the stochastic variables on consistency of results can be checked.

The second step is at the compartment level, when the different fragility functions FF_L are constructed. The local functions are generally different for each fire location within a same building as parameters such as the member sizes and fire exposure vary between compartments. This paper investigates how sensitive the FF_L are to the fire location within the building.

The third step is at the building level, with the building fragility functions FF_B. Within a building typology, the buildings have similar attributes but they are not identical. Parameters such as the building height, the building occupancy or the fire resistance rating of the building may vary. Therefore, it is crucial that the characterization of the fire performance take into account the variability in these parameters. In this study, the effect of these parameters on the fire performance is investigated by analyzing their effect on the fragility functions for the building, FF_B.

The paper thus presents a sensitivity analysis of fire fragility functions, which allows discussing the effects of various parameters on the global fire safety of buildings. This also allows identifying the parameters that must necessarily be considered as random in a probabilistic fire engineering analysis, at different scales. Finally, the methodology is useful to compare the fire vulnerability of different buildings within a typology (based on the FF_B), or different compartments within a building (based on the FF_L). These sensitivity analyses are presented in Sect. 4.

2.3. Fire Risk Assessment Framework

The building fragility functions characterize the overall vulnerability of the building to fire. Developed at the system scale, they implicitly account for any possible fire location. Yet, the functions yield a conditional probability to reach a predefined damage state (DS) as a function of the fire intensity as measured by an Intensity Measure (IM). In a reliability framework, one is usually interested in estimating a total (for instance annual) probability of failure. Therefore, a method is proposed to estimate the fire risk using fragility functions. This method is illustrated in Fig. 1 and applied in details on a prototype building in Sect. 5.

First, the dependence on the IM can be eliminated if one adopts a probability density function (PDF) for the IM, $f_q(q)$. The latter will usually depend on the building occupancy. Convolution of the fragility functions with this IM probability distribution yields a single value (scalar) for conditional probability of failure, $p_{F|H,fi}$. This probability is still conditional, it does not account for the probability that a fire break out in the building. Yet it is possible to account for this if the probability of occurrence of a fire in the building is estimated ($p_{H,fi}$). This probability depends on the building occupancy and size. Multiplication of the probability of occurrence $p_{H,fi}$ by the conditional probability of failure $p_{F|H,fi}$ results in a total probability of failure ($p_{F,total}$) for the entire building, for the considered DS. In other words, this yields the probability that the damage state will be reached somewhere in the building, due to fire, over the period considered (e.g. per year).

3. Application

3.1. Building Prototypes and Damage State

The method presented in Sect. 2 is applied to an illustrative example. The selected typology is a multi-story steel frame building (MSFB). Within this typology, building prototypes of variable heights, occupancy and fire resistance rating are considered. Four building heights are selected corresponding to 3, 6, 9, and 12 stories, which allows considering one structure in low-rise category (3-story), one structure in mid-rise category (6-story), and two structures in high-rise category (9 and 12 story), according to the *Building Structure Categories* for steel frames defined in Hazus [36]. Design of the prototypes is based on the FEMA/SAC project for the Los Angeles area [37]. They all have a 45.72 m by 45.72 m plan area, consisting of five bays of 9.14 m (30 ft) in both directions, as shown in Fig. 2. The structure is composed of four moment resisting frames on the perimeter, and interior gravity frames. The elevation of the gravity frames in the prototype buildings are shown in Fig. 2.

Fire can lead to various degrees of damage in buildings. Examples of damage states include cracking of a non-structural elements, excessive deflection, or failure of a structural element. In a resilience framework, the damage states should be

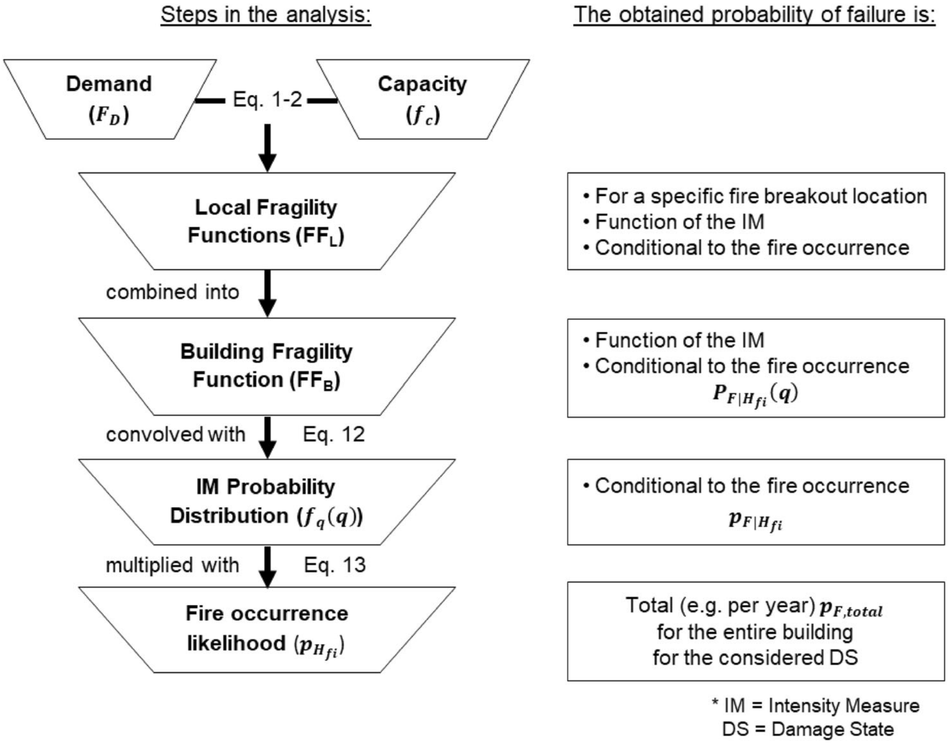

Figure 1. Method to assess the probability of failure (i.e. reaching a predefined damage state) due to fire for multi-story buildings, using fragility functions.

categorized as a function of their effect on the building's functionality. A fragility function needs to be derived for each damage state, to quantify the probability of exceeding the damage state. This example is presented for a damage state defined as the failure of an interior frame column. The columns of the interior frames are gravity columns with pinned connections to the beams. As there is no continuity in the horizontal beams, thermal expansion of the interior columns is not restrained. Hence, indirect actions owing to thermal restraints do not play a significant role for the structure and specific damage state under consideration. The columns are continuous for their full height. As they are interior columns, in the fire situation they are assumed to be heated on four sides.

To work on examples representative of real-life designs, current practice and codes were followed for structural design of the prototypes. Table 1 gives the sections obtained from design at ambient temperature. The column sections range from W14×43 to W14×145. The beams are composite with a steel deck and a concrete slab of 10.16 cm (4 inches) thickness. Table 2 gives the unfactored gravity loads distributed on the interior beams. The nominal values of the steel yield

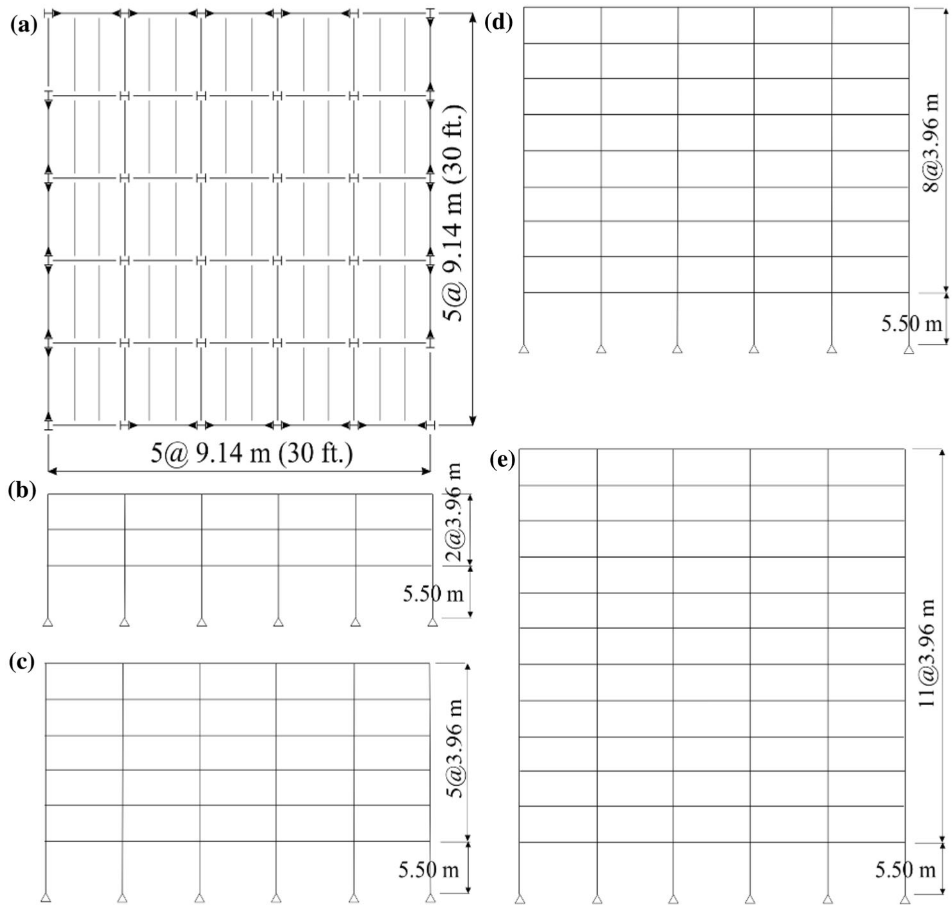

Figure 2. Steel gravity frames used as building prototypes for the fragility assessment (a) plan, and elevation of (b) 3-story, (c) 6-story, (d) 9-story, (e) 12-story.

strength and modulus of elasticity are 345 MPa and 200,000 MPa, respectively. The nominal value of the concrete compressive strength is 28 MPa.

For fire design, a prescriptive guideline was adopted. It is assumed that the buildings are sprinklered. First, the International Building Code [38] was used to obtain the required fire resistance rating for the building structural elements. For example, the 9-story steel building requires a 2-h fire rating for beams and columns in the frame. Then, Underwriters Laboratory (UL) publications were used to find the thickness of spray fire protection to apply on the elements, for the abovementioned fire rating. For the columns in this study, the fire protection is designed based on the X829 UL configuration (using a simple formula based on the ratio of column weight to heated perimeter to adjust between the configuration specified in UL publications and the element under study). The fire protection

Table 1
Section Design for the Prototype Buildings of Different Height

	Beam	Gravity column
3-story		
2-RF	W18×40	W14×43
1	W21×44	W14×53
6-story		
RF	W18×40	W14×43
4–5	W21×44	W14×53
2–3	W21×44	W14×68
1	W21×44	W14×90
9-story		
RF	W18×40	W14×43
7–8	W21×44	W14×53
5–6	W21×44	W14×68
3–4	W21×44	W14×82
1–2	W21×44	W14×109
12-story		
RF	W18×40	W14×43
10–11	W21×44	W14×53
8–9	W21×44	W14×68
6–7	W21×44	W14×82
4–5	W21×44	W14×109
2–3	W21×44	W14×132
1	W21×44	W14×145

The profiles used for the interior beams and gravity columns are given at the different stories (RF = roof)

Table 2
Gravity Design Load for the Structural Members

	Distributed loads (kN/m)		
Level	Dead load	Live Load	Partitions
Roof level	43.72	5.25	3.96
Other levels	41.88	13.21	3.96

material is a dry mix CAFCO Blaze-Shield II from Isolatek, with product specification and design aid provided by the manufacturer [39]. Table 3 shows the thickness of fire protection corresponding to the nominal design of 2-h rating, as well as for 1-h and 3-h ratings. The different fire rating designs are provided because the sensitivity of fire fragility curves to fire protection will be investigated.

Gypsum plasterboard is assumed as the lining material for walls and ceiling of the prototype building, with the following properties [40]: thermal conductivity $k_g = 0.48$ W/mK; specific heat $c_g = 840$ J/kgK; and density $\rho_g = 1440$ kg/m^3.

Table 3
Thickness of Fire Protection for the Gravity Frame Columns

		Thickness of fire protection (cm)		
Section size	W/D	1 h	2 h	3 h
W14×43	0.75	1.75	3.65	4.76
W14×53	0.91	1.75	3.18	4.29
W14×68	1.04	1.59	3.02	4.29
W14×82	1.23	1.43	2.70	3.13
W14×109	1.29	1.43	2.70	3.97
W14×132	1.56	1.11	2.38	3.49
W14×145	1.64	1.11	2.22	3.33

3.2. Probabilistic Models for Fire Development

The modeling of the fire development is the first stage in the analysis of the fire performance of a structural element. This study assumes that the fire can be modeled using a compartment fire model, i.e. it represents a post-flashover and single zone situation. It is also assumed that the fire remains contained in the compartment where it started (the possibility of fire spread across different compartments is out of the scope of the present study). The selected fire model is the widely used Eurocode 1 parametric fire model [41]. The original values of 0.1×10^{-3} and 0.20×10^{-3} are replaced with 0.14×10^{-3} in the equations A.7, A.9, and A.12 of Appendix A in Eurocode, to avoid the gap in temperatures when switching from fuel-controlled to ventilation-controlled fires. This modification, in accordance with Reitgruber et al. [42], has been calibrated on 50 full-scale tests and confirmed by further studies [43].

Fire load is the most important parameter in the analyses as it is used as the intensity measure for fragility functions. All analyses are conducted for several levels of fire load successively fixed between 100 MJ/m^2 and 2000 MJ/m^2, to cover the range of realistic fire loads in a building compartment. There is no need to adopt a probabilistic distribution for the fire load when constructing the fragility functions (by definition of the intensity measure). Given the fragility functions, the user can then assume the distribution of fire load in the building under study, based on the usage and other features of this building, in order to determine the probable level of damage. This could be done for instance using the NFPA 557 standard [44] or EC1 [41] for fire loads in buildings.

In the EC1 parametric fire model, the compartment size and opening factor are selected as random parameters. The nominal fire compartmentation of the building prototypes is assumed to be based on a subdivision in compartments of 9.144 m long (30 ft), 6.096 m wide (20 ft) and 2.8 m (9 ft) high, i.e. a floor area of 55.74 m^2. The openings for the nominal compartment have a width of 3 m and height of 1.5 m. For sensitivity analysis, the compartment size varies between 5 m to 10 m in length, 3 m to 8 m in width, and 2.5 m to 3.2 m in height. All three dimensions are assumed to follow a uniform distribution, i.e. all values in a given interval are equally probable. Therefore, the considered compartment areas are in the range of 15 to 80 m^2. Each time that the compartment dimensions change, the size of openings are also adjusted proportionally based on the ratios assumed in the nominal values of the compartment and opening sizes. The opening factor is calculated according to the formula provided by EC1:

$$O_{max} = \frac{A_v \sqrt{h_{eq}}}{A_t} \tag{3}$$

where A_v is the area of the vertical openings on all walls, A_t is the total area of enclosure, and h_{eq} is the weighted average of window heights on all walls. The opening factor from Eq. 3 provides the maximum possible value assuming that window glass is immediately broken when fire breaks out. JCSS [45] models the value of opening factor as a random quantity according to:

$$O = O_{max}(1 - \zeta) \tag{4}$$

where O_{max} is calculated from Eq. 3 and ζ is a random variable that follows a truncated lognormal distribution with mean 0.2 and standard deviation 0.2 (truncated implies values are cut off at $\zeta = 1$). Equation 4 is used to incorporate randomness in the value of opening factor.

3.3. Probabilistic Models for Thermal Response

Knowing the gas temperature evolution in the compartment, the next stage consists of the heat transfer analysis in the section of the column. For heat transfer analysis, the finite difference formula of Eurocode 3 part 1–2 is adopted [46]. Since the example consists in an interior column heated on four faces, the Eurocode formula is deemed as a reasonable method. Analytical approaches are preferred in this work because they can be implemented and run at very low computational cost, compared to advanced methods such as finite element method (FEM). Indeed, a large number of realizations are required for the MCS to provide a good estimate of the probability of failure. This formula, also referred to as lumped mass approach, yields the uniform temperature in the cross-section of a steel member at each time step and it can be used for insulated (Section 4.2.5.2 in [46]) and bare (Section 4.2.5.1 in [46]) steel members. This formula is used to get the maximum temperature T_{max} reached in the section during the course of the natural fire; this maximum temperature is the demand placed on the element (see F_D in Eq. 1).

The formula of EC3 depends on the section factor; the thickness, specific heat, thermal conductivity and density of the insulation material; the specific heat and density of steel. Generally, fire protection manufacturers provide certain properties of their products, including density and thermal conductivity at ambient temperature. Many studies in the literature use the values provided by the manufacturer at ambient temperature for the whole duration of fire, since there is limited data to describe the change in properties of sprayed fire resistive materials (SFRM) at elevated temperatures. However, NIST [47] performed a study on passive fire protection for buildings, in particular the World Trade Center buildings, and presented test results for properties of insulating materials at high temperatures (density, thermal conductivity, and specific heat). The NIST study tested three SFRM. Properties of these materials varied from type to type and with varying temperature. The data shows that as temperature increases, thermal conductivity and specific heat increase while density decreases. Elhami Khorasani et al. [17] used the data from the NIST study, and a Bayesian based approach, to develop probabilistic models for density (ρ_i), thermal conductivity (k_i), and specific heat (c_i) of SFRM; see Eqs. 5–7 where T is in Celsius and ε is a random variable with standard normal distribution. These equations are used to incorporate uncertainties in the thermal properties of SFRM. It must be stressed that it is unconservative to assume a constant value of thermal conductivity equal to the value at ambient temperature, since the NIST data shows an increase in thermal conductivity with temperature.

$$k_i = \exp\left(-2.72 + 1.89 \times 10^{-3} T - 0.195 \times 10^{-6} T^2 + 0.209 \times \varepsilon\right) \tag{5}$$

$$\rho_i = \exp\left(-2.028 + 7.83 \times T^{-0.0065} + 0.122 \times \varepsilon\right) \tag{6}$$

$$c_i = 1700 - \exp\left(6.81 - 1.61 \times 10^{-3} \times T + 0.44 \times 10^{-6} T^2 + 0.213 \times \varepsilon\right) \tag{7}$$

The thickness of insulation during the heat transfer analysis is also an important factor that generally has a relatively large uncertainty given the method of application. A lognormal distribution is assumed for this parameter with a mean equal to the nominal value (as designed and listed in Table 3) plus 1.6 mm, and a coefficient of variation of 0.2 [48].

The section factor depends on the geometry of the element section. Uncertainty in the element is taken into account by considering various fire locations in different prototype buildings (since the element section depends on the prototype and story, see Table 1). Hence, the fire performance of different sections is compared. Yet, for a given section type, the section factor is taken as deterministic, i.e. assuming that the manufacturing process for steel sections is well controlled with very limited uncertainty in the dimensions. Similarly, the specific heat and density of steel are considered as deterministic, given the relatively low variances expected for these parameters [18]. These are taken in accordance with Eurocode 3 part 1–2 including the temperature dependency of the specific heat of steel.

3.4. Probabilistic Models for Structural Response

The structural analysis aims at capturing the column failure at high temperature. Similar to the fire and thermal models, a simplified method is adopted here, namely the simple calculation model from EC3 [46] where the temperature dependent buckling resistance of the column $N_{b,T,Rd}$ is given by:

$$N_{b,T,Rd} = \chi_{fi} A f_{y,T} \tag{8}$$

with χ_{fi} the reduction factor for flexural buckling in the fire situation, A the cross-section area, and $f_{y,T}$ the yield strength of steel at temperature T, assumed uniform in the cross-section. This model allows calculating the resistance of a compression member at high temperature taking into account flexural buckling. It is obviously a simplified model based on conservative assumptions. An important assumption is that the cross-section is at a uniform temperature. For the protected steel columns heated on four sides that are studied here, this assumption is reasonable. Another simplification is that the column is subjected to compression only and the compressive applied load remains constant during the fire. As this study deals with gravity columns with pinned beams, this assumption is deemed acceptable. Finally, this model does not account for local buckling so it should not be used with slender sections. Selection of a simplified model is motivated by the constraints associated with estimating a very large number of realizations in the brute force MCS approach used here. Advanced models such as FEM are commonly used for capturing the behavior of entire structural assemblies under fire in a deterministic framework, but these are computationally very expensive. Research is underway for developing probabilistic methods to allow efficient estimation of probability distributions using a limited number of model evaluations [49], which would allow use of FEM in a probabilistic framework.

In estimating χ_{fi}, the moment of inertia corresponding to the member's weak axis is selected. The critical temperature $T_{critical}$ at which failure is reached is obtained by solving the equation for $N_{b,T,Rd}$ equal to the applied axial load on the column. The critical temperature is the capacity of the element (see f_C in Eq. 1). This critical temperature depends on the column geometry, slenderness, applied loads, and steel mechanical properties.

Uncertainties in steel properties are considered using the probabilistic models developed by Elhami Khorasani et al. [17], in particular the EC3-based logistic expression of Eq. 9 in [17] for the steel yield strength retention factor and the logistic (no-base) expression of Eq. 13 in [17] for the steel modulus at high temperature. These models allow randomly generating the yield strength and modulus of elasticity as a function of temperature. The models are based on a large pool of measured data that exist in the literature and a Bayesian based formulation to find the best fit and the model errors. As the dataset includes data at ambient temperature, initial variability in yield strength and modulus at ambient temperature is also accounted for. In this paper, it is assumed that the yield strength and modulus of elasticity are correlated, implying that, for instance, if the yield strength is one standard deviation above the median, the same holds true for the modulus.

Uncertainties in applied loads are also considered. The probability of having the maximum live load in the structure during a fire event is relatively low; therefore, the applied loads during fire are defined as a fraction of the design loads. Following the approach by Ellingwood [50] and Iqbal and Harichandran [48], the applied load P is modeled as:

$$P = E\left(AP_{DL,fi} + BP_{LL,fi}\right) \tag{9}$$

where $P_{DL,fi} = 1.05 \times P_{DL}$, $P_{LL,fi} = 0.24 \times P_{LL}$, P is the total gravity load, and A, B and E are factors to account for variability in the loads. Table 4 shows the assumed distributions for the parameters in defining the total gravity load P. The nominal value for dead load is the sum of dead and partition loads from Table 2. Finally, different column types are studied for the different building prototypes and fire locations. For a given column, the geometry of the section and slenderness are assumed to be deterministic parameters with the nominal values.

3.5. Local Fragility Functions

In the prototype multi-story building, the location of the fire is a priori unknown. Depending in which compartment the fire develops, the dimensions and section type of the structural elements may vary. Therefore, the uncertainty in these parameters must be considered when assessing the building vulnerability to fire.

In this approach, for the frame column damage state, the different configurations (column height, section type) corresponding to different possible fire locations in the building are successively studied, using the nominal values for each. The section types listed in Table 1 are considered with the column height corresponding to the design from Fig. 2. Fragility functions are derived for each configuration. Because those fragility functions are associated with a well-defined location of the fire, they are referred to as local (FF_L). In the sensitivity analysis, FF_L corresponding to different fire locations are compared to investigate whether structural fire reliability is evenly distributed in the building or there exists 'weak points' i.e. compartments where fire is more likely to trigger column damage.

3.6. Building Fragility Functions

The variability in building height within the MSFB typology is accounted for by studying four prototypes of 3, 6, 9, and 12 stories. The prescriptive guidelines from the IBC code [38] require the following, when not accounting for the presence of sprinklers: 3-h fire rating for the 12-story building, and 2-h fire rating for the other buildings (low- and mid-rise). The same prescriptive guidelines allow reducing the required fire rating to account for the beneficial effect of fire protec-

Table 4
Probabilistic Models for the Parameters Used in the Determination of the Applied Load

Parameter	Distribution	Mean	COV
Dead load [N]	Normal	1.05 × Nominal	0.1
Live load [N]	Gamma	0.24 × Nominal	0.6
A factor	Normal	1	0.04
B factor	Normal	1	0.2
E factor	Normal	1	0.05

tion measures such as sprinklers. For the buildings under consideration, accounting for the presence of sprinklers leads to the following minimum requirements: 2-h fire rating for the 12-story and 9-story buildings, 1-h fire rating for the 6 story building, and no requirement for the 3-story building. In this study, for comparison purposes, fire resistance ratings ranging from no insulation to 3-h fire resistance insulation are considered, which translates into different insulation thicknesses based on prescriptive design (Table 3). Note that the case with no insulation is also potentially relevant for taller buildings in a cascading multi-hazard scenario, e.g. after an earthquake or a blast that would damage the insulation. The sensitivity to building occupancy is also investigated. Comparison of building fragility functions for various combinations of building height, fire resistance rating and occupancy allows discussing the effects of these parameters on the vulnerability to fire.

4. Sensitivity Analysis

4.1. Demand: Fire and Thermal models

This section investigates the effect of randomness in the parameters of the fire and thermal models on the PDF of the maximum temperature reached in the column section T_{max}, i.e. the demand on the element. The probabilistic analyses are performed using MCS. The number of realizations in the MCS (size of the samples) is determined using a stopping (convergence) criterion corresponding to a coefficient of variation of 3% of the simulated mean temperature. The number of realizations ranges between 200 and 1000 depending on the number of random variables in the MCS.

Figure 3 shows the range of gas temperature–time curves obtained for fire loads q = 400 and 800 MJ/m^2, using the EC1 fire model with uncertainties in compartment size and opening factor. The fire load influences significantly the severity of the fire, both in terms of the maximum gas temperature and duration of the fire. This confirms the selection of fire load as the intensity measure for characterization of fire hazard in fragility analysis. For a given fire load, randomness in compartment size and opening factor also affect the fire, especially in terms of duration.

Figure 4 shows the complementary cumulative distribution (i.e. exceedance function) of maximum temperature in a W14×68 section column with a 2-h fire protection exposed to the fire on four faces. The results are given for fire loads of 400, 800 MJ/m^2 and 1200 MJ/m^2. Uncertainties in compartment size, opening factor, fire protection thickness and fire protection thermal properties are included. Functions corresponding to normal distributions (with the mean and standard deviation computed from the MCS results) are plotted next to the results of MCS. The maximum temperature in the steel section follows approximately a normal distribution. For a given section with a given thermal protection, the fire load influences significantly the maximum steel temperature reached in the element (demand). The computed distributions of maximum temperature in the steel section are in line with studies by others. For instance, Law et al. [51] provided results for

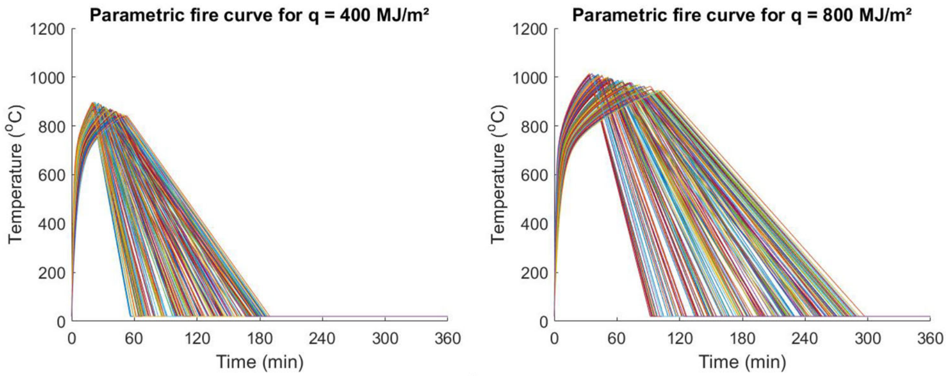

Figure 3. Gas temperature–time relationships generated by MCS based on the EC1 parametric fire model with randomness in compartment size and opening factor.

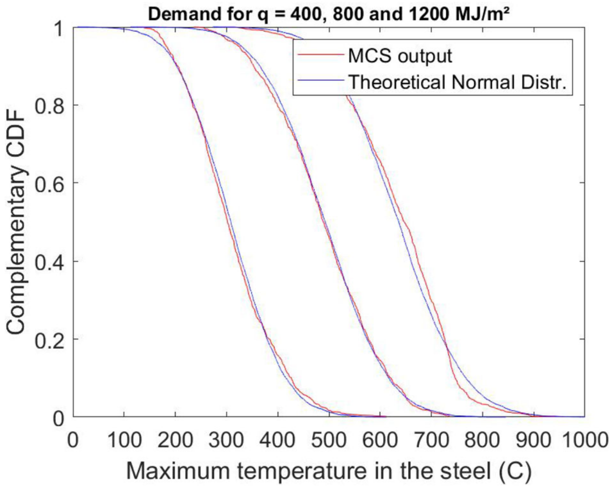

Figure 4. Distribution of maximum steel temperature reached in a W14×68 steel section protected with a 2-h rating and exposed to the fire on four faces. The distributions are generated by MCS considering randomness in compartment size, opening factor, fire protection thickness and fire protection thermal properties. Steel temperatures increase with fire load.

a notional steel section, where the objective was to determine the amount of fire protection to apply for a given target reliability. Using a similar lumped mass heat transfer approach and adopting a range of possible design fires for an office building, with the fire load distribution based on EN1991-1-2 (average of 420 MJ/m^2), they found that a 88 min fire protection was required to prevent the steel from exceeding a limiting temperature of 550°C at the 90th percentile. This is in line with Fig. 4 where a 120 min fire protection yields a steel temperature lower than 550°C at the 99.8th percentile for a 400 MJ/m^2 fire load and at the 73th percentile for a 800 MJ/m^2 fire load.

In Fig. 4, the observed variability in maximum steel temperature results from uncertainties in several parameters. In the following, a variance-based sensitivity analysis is conducted to gain further understanding of the effect of each single parameter, i.e. the effect of the variance of each input parameter on the variance in the output is isolated. This is completed by conducting a set of MCS, where a nominal value equal to the mean value is used for all input parameters except for the parameter of interest, for which the values are selected randomly based on the probability distribution (i.e. all parameters are deterministic except one). For instance, for analyzing the effect of uncertainty in fire protection conductivity, a large number (defined by the convergence criterion) of simulations are run in which fire protection conductivity is treated as a random parameter while the other parameters are kept deterministic. Therefore, the variability in maximum steel temperature obtained from this analysis results entirely from the effect of uncertainty in fire protection conductivity. The same procedure is then applied successively to the other input parameters.

Figure 5 presents a sample of results for column sections W14$\times$68 and W14$\times$109, respectively, protected with a prescriptive 2-h fire rating and exposed on four faces. The plots show the mean, plus and minus one standard deviation, of the maximum steel temperature for different selected random parameters. The results are given for fire loads equal to 400 MJ/m^2 and 800 MJ/m^2. The two values of mean and standard deviation provide a reasonable measure of uncertainty since the results follow approximately a normal distribution. Results for other section types, fire loads and fire resistance rating are consistent.

The parameters ‘compartment size’ and ‘opening factor’ influence the fire model, whereas the parameters related to fire protection influence the thermal model. The results show that uncertainties in both the fire model and the thermal model cause significant variance in demand. Hence, it is important to adopt probabilistic models for both models. In the thermal model, the thickness and conductivity of fire protection cause much larger variance compared to the density and specific heat of fire protection. Therefore, density and specific heat of fire protection can be treated as deterministic. In case of thermal properties of fire protection, although modeling approaches are different, the results are consistent with those reported by Guo and Jeffers [18].

4.2. Capacity: Structural Model

This section investigates the effect of randomness in the parameters of the structural model. The output of the structural model is the critical temperature at which the column fails $T_{critical}$, i.e. the capacity of the element.

Figure 6 shows the distribution of critical temperature for a W14$\times$68 section column obtained from MCS. The column capacity does not depend on the characteristics of the fire (such as the fire load). However, it depends on the story level, because the story influences the load on the column. The results are given

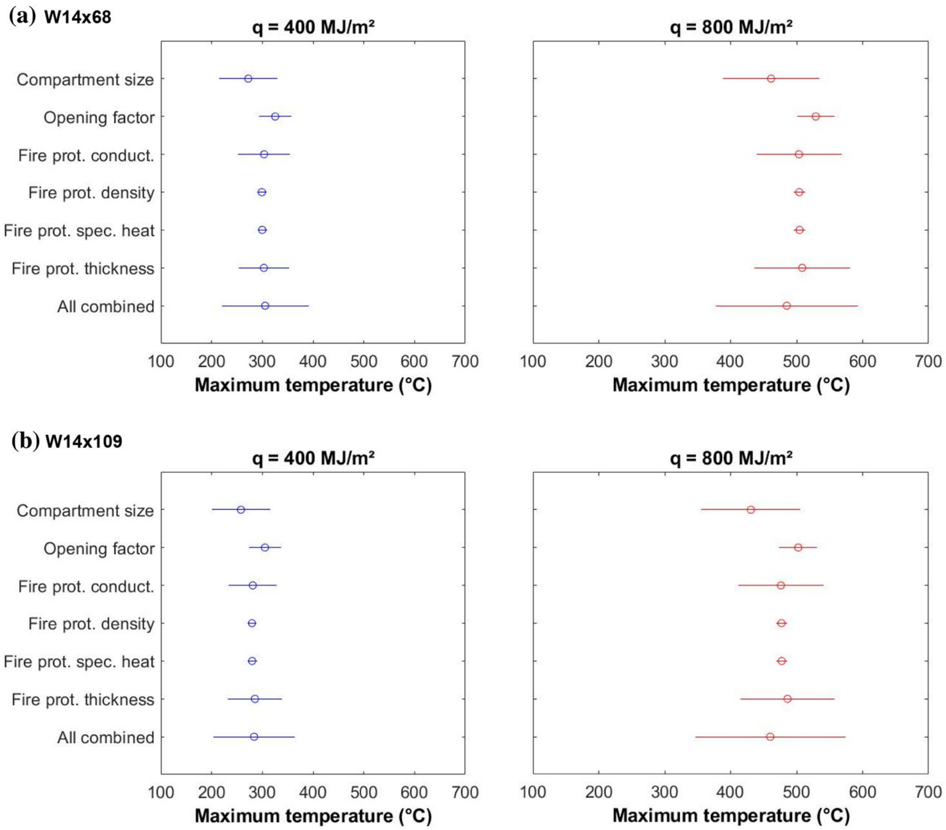

Figure 5. Sensitivity of maximum steel temperature to demand parameters for (a) W14×68 and (b) W14×109 sections protected with a 2-h rating and exposed to the fire on four faces. The plots show a width of one standard deviation around the mean value.

for the columns at the sixth story of the nine-story building. Uncertainties in steel mechanical properties, dead load and live load are included. The critical temperature of the column follows approximately a normal distribution.

The effect of the variance of each input parameter is then studied separately, as was done for the demand. Figure 7 presents a sample of results for column sections W14×68 and W14×109, respectively. The plots show the mean, plus and minus one standard deviation, of the critical steel temperature for different selected random parameters. As the same column section type is used in two successive stories in the design, the results are given for the two stories. For a given section type, the critical temperature is higher for the upper story because the applied loads are lower. Results for other section types are consistent.

The randomness in steel mechanical properties (yield strength and modulus of elasticity) at elevated temperature contributes the most to the variance of the capacity. In contrast, the influence of live load is negligible. This result may seem

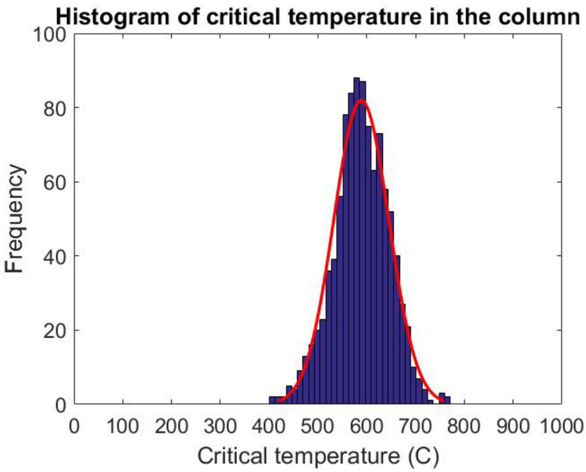

Figure 6. Distribution of critical temperature for a W14×68 column at the 6th story of the 9-story building. The distributions are generated by MCS considering randomness in steel mechanical properties, dead load and live load.

counter-intuitive, but it is due to the small ratio of live load to dead load (Table 2) and the relatively limited variance of live load (Table 4) for the studied prototypes. Therefore, live load could be considered as deterministic in this study. A previous study [18] also confirmed the sensitivity of results to both yield stress and modulus of elasticity, but only assumed randomness in yield stress.

4.3. Local Fragility Functions

Assessment of the probabilistic demand and capacity of the elements allows constructing the fragility curves using Eqs. 1 and 2. The fragility curves yield the probability of reaching a damage state conditional to the occurrence of a fire H_{fi}. The local fragility functions FF_L are different for each location of the fire within a prototype building. In constructing the FF_L, randomness is included in the demand and capacity parameters identified in the previous sections as being significant sources of uncertainty. This means that compartment size and opening factor (fire model), fire protection conductivity and thickness (thermal model), steel mechanical properties and dead load (structural model) are treated as random parameters in the analyses. The FF_L are built independently for different column section and column height. Then, these FF_L are compared to analyze the influence of the compartment where fire occurs on the vulnerability of the frame columns.

Each FF_L is constructed using 20 values of the fire load (intensity measure), evenly distributed between 100 MJ/m^2 and 2000 MJ/m^2. For each fire load, the probability of failure is estimated using Eq. 1, in which MCS are run to derive the distribution of demand $F_D(T, q_i)$ and the distribution of capacity $f_C(T)$. Note that the distribution of capacity needs only be derived once as it does not depend on

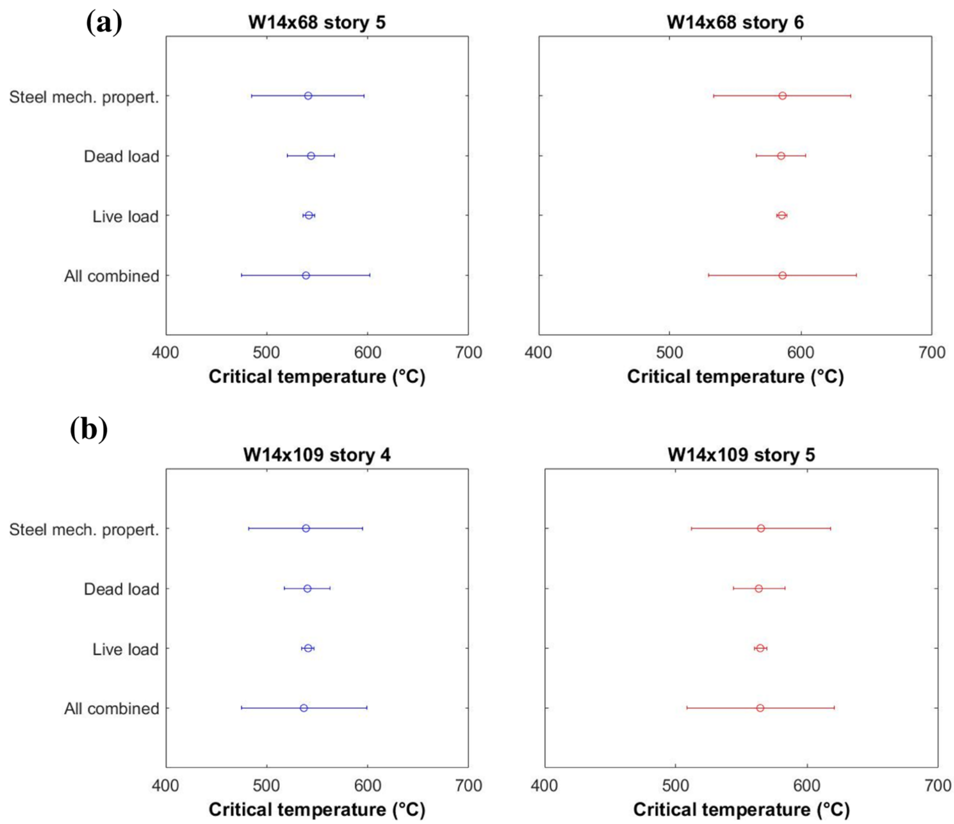

Figure 7. Sensitivity of steel temperature at failure to capacity parameters for (a) W14×68 in the 9-story prototype and (b) W14×109 section in the 12-story prototype. The plots show a width of one standard deviation around the mean value. Failure refers to the considered damage state, i.e. here the ultimate strength state (buckling) of the column.

the fire load q. In contrast, for each column, the distribution of demand is evaluated for each considered value of the fire load (i.e. 20 different distributions of demand are derived).

Figure 8 compares the FF_L for different column section types along the height of the 12-story prototype building (i.e. effect of the story level on FF_L). The figure shows the fragility points and fragility curves match well, confirming the validity of lognormal assumption in Eq. 2. The plot shows that the column of the first story is the most vulnerable in this building. This can be explained by the higher slenderness ratio (larger story height and the pinned boundary condition at the base while intermediate stories have rotational stiffness on both ends coming from the cold upper and lower columns). When designing the building, the utilization ratio of columns along the height is optimized and approximately kept the

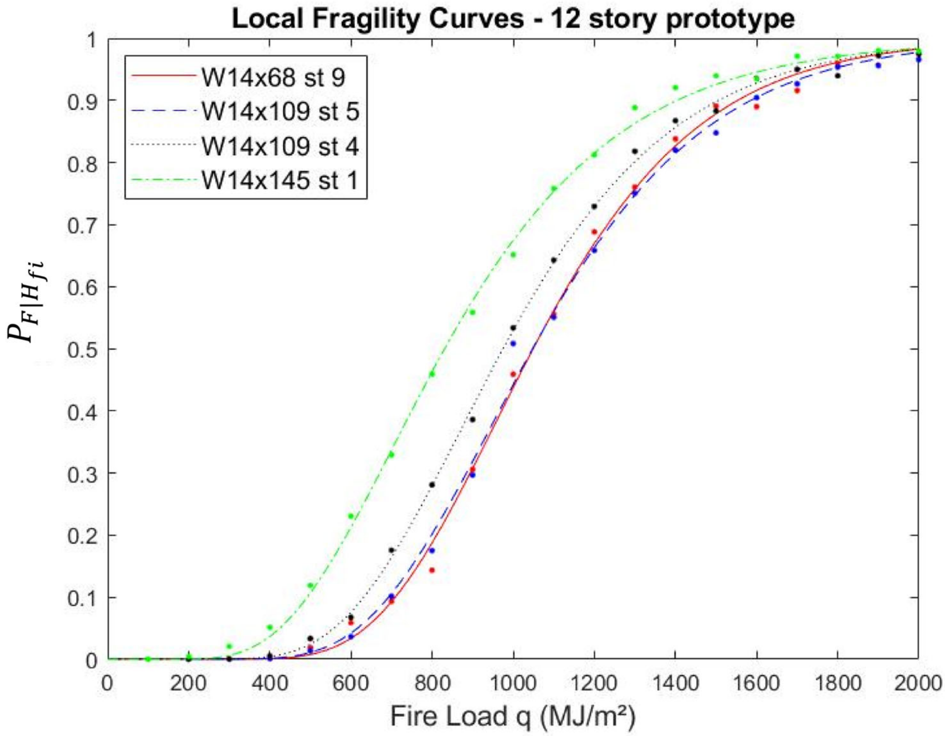

Figure 8. Fragility points (computed) and fragility curves (lognormal fit) at the local level (FF_L) for the fire-induced failure of an interior frame column, with the fire load as the hazard intensity measure. Fragilities are given for different columns of the 12-story prototype assuming 2-h rating protection.

same to obtain a similar safety level at each story. Meanwhile, when the same column section is used on two stories (column splices are located at every two story), the lower story is more vulnerable than the upper because of larger gravity loads on the lower story.

4.4. Building Fragility Functions

This section investigates the parameters influencing the fragility function at the building scale (FF_B), in particular, building height, fire resistance rating, and occupancy type. The fire resistance rating is considered at the building scale, because in current practice, the code prescribes the required fire rating for the entire building. The fragilities for the buildings (FF_B) are determined by combining the individual fragilities at the compartment scale (FF_L).

Figure 9 shows the FF_B obtained for different fire resistance ratings and building heights. The damage state is a failure of an interior frame column heated on four sides. The results show that the building height does not influence signifi-

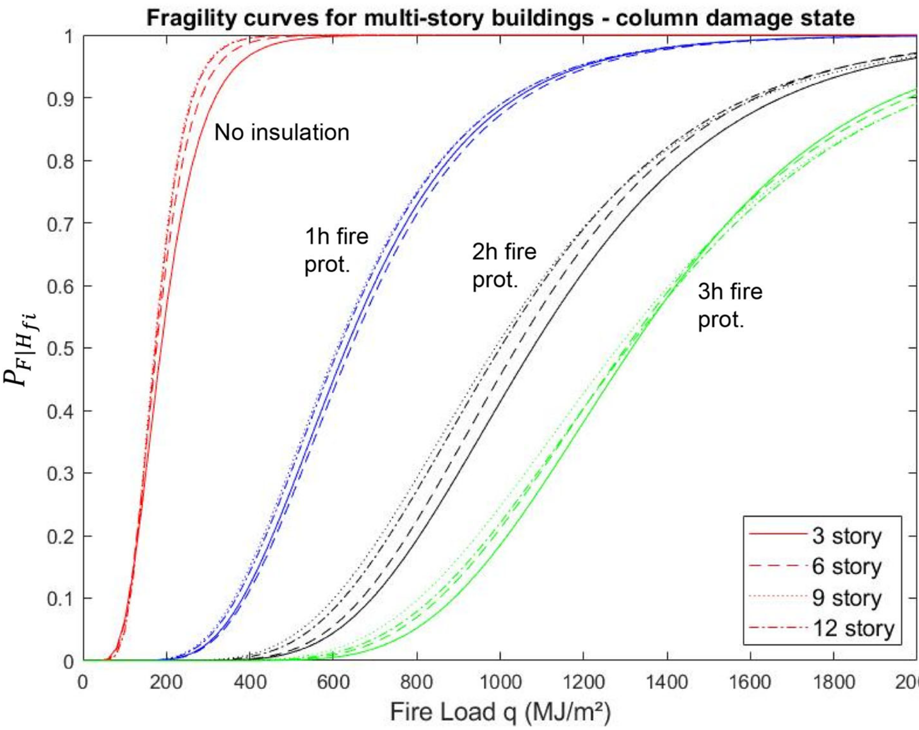

Figure 9. Fragility curves at the building level (FF_B) for the prototype multi-story buildings, for the damage state corresponding to the failure of an interior frame column. Fragilities are given for different building heights and fire resistance ratings.

cantly the fragility function. This implies that, given a fire in the building, the probability of reaching the column damage state is similar regardless of the height of the building. Indeed, columns are designed for a consistent utilization ratio at every stories.

The different fire resistance ratings considered in Fig. 9 range from no insulation to 3-h fire rating (see Table 3 in Sect. 3.1 for the corresponding fire protection thickness). As expected, the effect of fire resistance rating is significant and plays an important role in fire performance of the structure. Assuming a fire develops in a 12-story MSFB that contains an average fire load of 600 MJ/m^2, the probability that the fire-exposed frame column fails is 0.48 if the structure is protected with a 1-h fire protection rating; 0.08 for a 2-h rating; and it drops to 0.01 for a 3-h rating. Without any insulation, the structure has a probability virtually equal to 1 to lose the column. Alternatively, one can also determine the maximum fire load density in a building for a given (deemed acceptable) conditional probability of failure. For instance, for a 25% probability of failure, the maximum fire load in a MSFB without any protection should be limited to 140 MJ/m^2, whereas it could reach up to 1030 MJ/m^2 if the building is protected with a 3-h fire resistance rating.

The case without any fire protection has a much larger probability of reaching the column damage state for low values of fire load in comparison to the other three cases that have fire protection. The prescriptive guidelines from the IBC code require the 12-story building to have a 3-h fire rating, while the other buildings (low- and mid- rise) require a 2-h fire rating (when not accounting for the reduction in fire rating allowed due to the presence of sprinklers). The results show that the 12-story building (with 3-h fire rating) has a smaller probability of reaching the damage state, when compared to the shorter buildings (with 2-h fire rating). This finding does not imply that the probability of failure due to fire ($p_{F,total}$) is lower in the 12-story building, since the fragilities provide the probability of reaching the damage state given that there is a structurally significant fire in the building ($p_{F|H_{fi}}$). This still has to be weighted with the probability to have a fire in the building ($p_{H_{fi}}$), as will be further discussed in Sect. 5.

The sensitivity of building fragilities to building occupancy is also investigated. It is found that the fragility, i.e. conditional probability of failure, is virtually the same whether the building is used for offices, dwellings or others. In fact, the occupancy influences the probability to have a fire in a compartment, but it does not influence the vulnerability of the structure once a fire has occurred; this is discussed further in Sect. 5.

5. Fragility Functions in a Reliability Framework

5.1. From Conditional to Total Probabilities in Fragility Functions

Results in Sect. 4 show that buildings which are similar in terms of structural type, fire rating design and compartmentation but have a different number of stories (3 to 12) and occupancy can be characterized by the same fragility functions for the columns. This implies that, once a fire develops somewhere in the building, the structural vulnerability to fire is similar. Indeed, all columns in the buildings have approximately similar demand over capacity ratio (D/C) at ambient temperature and will have a similar fire response when the same prescriptive fire rating is applied.

However, fragility functions yield conditional probabilities, i.e. the likelihood of having a fire in the building is not accounted for in the functions. It is interesting to compute the total probability in order to take into account differences in propensity to have a fire in different buildings. The total probability (per year) of reaching a damage state (e.g. a column failure in this study) due to fire in a building, $P_{F,total}$, can be obtained by multiplying the conditional probability given by the fragility function FF_B, noted $P_{F|H_{fi}}$, (see Eq. 1) with the annual probability of occurrence of a structurally significant fire in the building, $p_{H_{fi}}$, see Eq. 10. The term $p_{H_{fi}}$ is a scalar whereas the two other terms in the equation are functions of fire load density (i.e. the intensity measure). Equation 10 amounts to scaling the fragility function by the probability of occurrence of a fire.

$$P_{F,total}(q) = P_{F|H_{fi}}(q) \times p_{H_{fi}} \tag{10}$$

The formula of Eq. 11 can be used to estimate $p_{H_{fi}}$ in a building with a total floor area of A_{fi} (m^2). This formula is the one used for the development of the fire load density design values prescribed in EC1 [41].

$$p_{H_{fi}} = p_{1,EN} \times p_{2,EN} \times p_{3,EN} \times p_{4,EN} \times A_{fi} \tag{11}$$

Here, the area of the prototype buildings is 2090 m^2 per story (related to A_{fi}). For the sake of discussion, a series of assumptions are made for the other factors. The term $p_{1,EN}$ is the probability of having a fire to start and grow to a severe fire in the compartment, per m^2 of floor and per year. For an office building, a value of 3×10^{-7}/m^2 year is adopted [52]. Additional reduction factors are then applied to the annual frequency to account for active fire protection measures. The factor $p_{2,EN}$ considers the effect of the fire brigade type and the time between alarm and firemen intervention. This value equals to 0.1 assuming a professional fire brigade with a required intervention time between 10 min and 20 min. The factor $p_{3,EN}$ takes into account the effect of automatic fire detection and automatic transmission of the alarm; assuming automatic fire detection by smoke leads to a value of 0.0625. Finally, the factor $p_{4,EN}$ takes into account the effect of sprinkler; this factor can be taken as 0.02 assuming normal sprinklers (e.g. according to the regulations). As a result, $p_{H_{fi}}$ varies from 2.4×10^{-7} for a 3-story building to 9.4×10^{-7} for a 12-story building. This probability represents only structurally significant fires, i.e. the ones that develop and grow to severe despite the possible action of sprinklers, occupants and fire brigades.

The total (annual) probability of column failure due to fire can be plotted by multiplying the fragility functions by the probabilities $p_{H_{fi}}$ in the corresponding buildings. Figure 10 shows these total probabilities for the buildings of different heights. The results are given for office buildings and columns exposed on 4 faces. For a meaningful comparison of the actual reliability level according to prescriptive guidelines, the fragility functions to be used in Eq. 10 are the ones corresponding to the fire rating required at each building height. Two design situations are presented, namely (a) the case where the IBC prescriptive design is followed without using the provision to reduce the fire rating owing to the presence of sprinklers, and (b) the case where this reduction is applied (see Sect. 3.6). The shape of the curves in Fig. 10 are consistent with the functions of Fig. 9, where the scaling of Eq. 10 has been applied. For very large values of the fire load for which the fragility curves converge towards one, the total probability curves converge towards the annual probability of having a fire in the corresponding building, $p_{H_{fi}}$. At constant fire rating, the probability of failure increases with the number of story when total probability (not conditional) is considered, see Fig. 10a. This reflects the higher likelihood of having a fire in a taller building (e.g. 9 story) compared with a low-rise one (e.g. 3 story). However, when the fire rating is adjusted to account for the presence of sprinklers, the relationship between $P_{F,total}(q)$ and the number of stories is more complex, see Fig. 10b. Taller

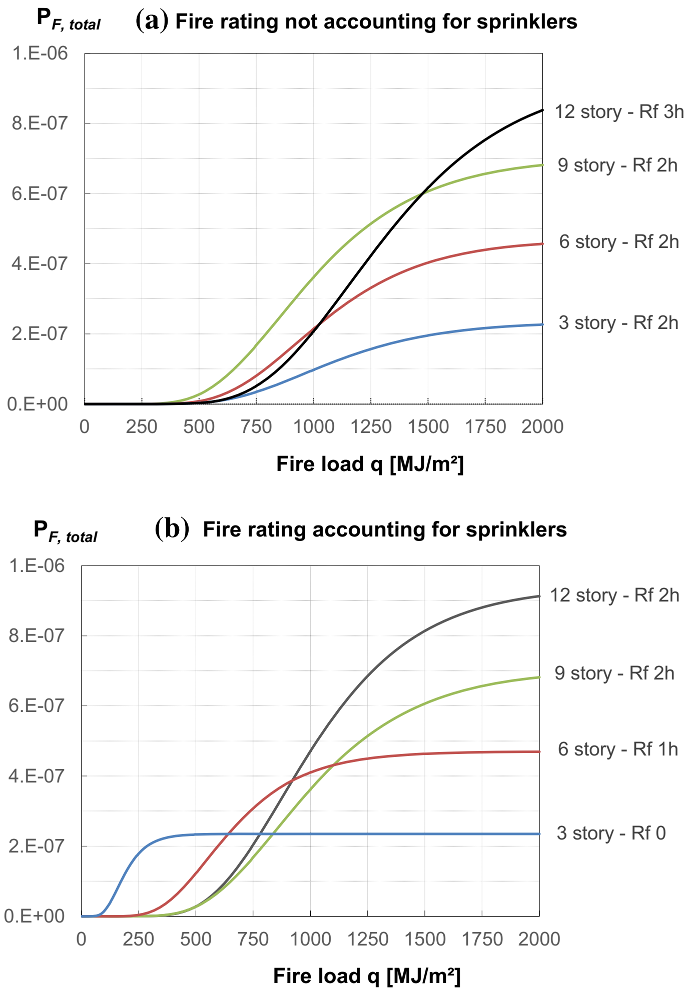

Figure 10. Probability of failure of an internal frame column per year due to fire for sprinklered multi-story office buildings of different heights and prescriptive fire rating, as a function of the fire load in the building. The selected fire rating design is based on the IBC prescriptive requirement (a) without reduction for sprinklers; and (b) with reduction for sprinklers.

buildings show better performance at low fire load levels ($<$ 750 MJ/m^2), as increased thermal protection prevents failure to occur under weak fires. In contrast, taller buildings have more likelihood to fail at large fire load levels ($>$ 1100 MJ/m^2) compared with shorter buildings, because the former have more likelihood to experience a fire and such severe fires cause failure of even highly protected columns. It is interesting to observe that accounting for the presence of sprinklers in the fire rating leads to a somewhat consistent reliability between

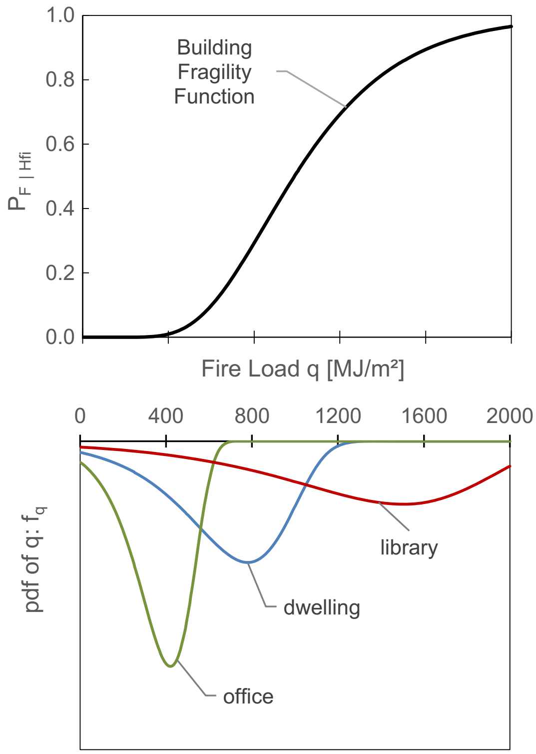

Figure 11. Fragility function as a function of the fire load (intensity measure) and probability distributions of the fire load densities for different building occupancies.

buildings of different heights in the range 700–900 MJ/m^2. However, this also leads to high tolerated probabilities of failure for low rise buildings at low fire loads (250–500 MJ/m^2) as the fragility is basically equal to unity for these unprotected structures.

The results have important implications in terms of structural reliability of buildings, implying that the prescriptive approach does not provide the same level of safety in different building heights in case of fire. When designing at the compartment level, the elements (e.g. the columns) are designed individually to have a certain degree of reliability (sometimes expressed in terms of reliability index). This is achieved, for instance, by requiring the columns to have a 2-h fire resistance rating according to prescriptive guidelines. However, this approach disregards the fact that the reliability at the building level depends both on the reliability of the building elements and on the number of elements that compose the building. In other words, the annual probability of failure of at least one column in a building increases if the number of columns in the building increases, assuming that the columns have all the same probability of failure. For the considered prototypes and damage state, this results in a higher probability of a sin-

Table 5
Application of the Method of Fig. 1 to the Case of the Prototype Buildings to Yield, from the Fragility Functions, the Annual Probabilities of Column Failure Due To Fire

No story	Fire rating (h)	Occupancy	FF_B (i.e. $P_{F\|H_{fi}}(q)$)		$f_q(q)$ (MJ/m^2)				
			c	ζ	μ	σ	$p_{F\|H_{fi}}$ (Eq. 12)	$p_{H_{fi}}$ (Eq. 11) $\times 10^{-7}$/year	$p_{F,total}$ (Eq. 13) $\times 10^{-7}$/year
3	2	Office	1078	0.344	420	126	0.006	2.4	0.01
3	2	Dwelling	1078	0.344	780	234	0.150	5.1	0.76
3	2	Library	1078	0.344	1500	450	0.561	2.4	1.32
6	2	Office	1039	0.344	420	126	0.008	4.7	0.04
6	2	Dwelling	1039	0.344	780	234	0.170	10.2	1.73
6	2	Library	1039	0.344	1500	450	0.584	4.7	2.74
9	2	Office	988	0.386	420	126	0.017	7.1	0.12
9	2	Dwelling	988	0.386	780	234	0.213	15.3	3.25
9	2	Library	988	0.386	1500	450	0.602	7.1	4.24
12	3	Office	1306	0.346	420	126	0.001	9.4	0.01
12	3	Dwelling	1306	0.346	780	234	0.070	20.4	1.43
12	3	Library	1306	0.346	1500	450	0.428	9.4	4.03

The values are given for different building heights and occupancies

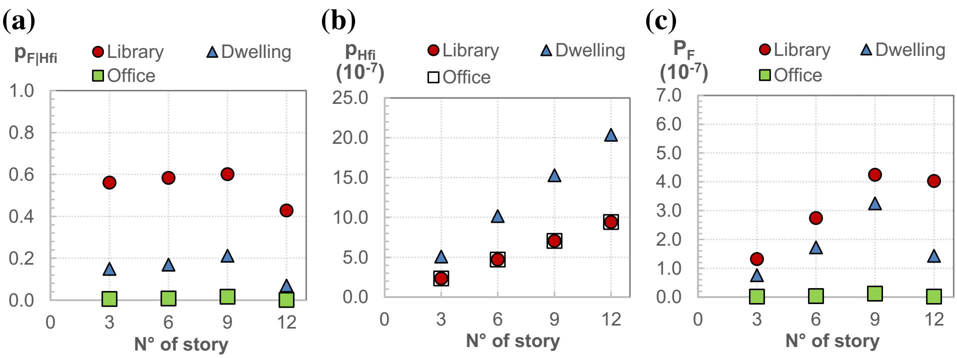

Figure 12. Comparison of fire reliability levels for sprinklered MSFB with different heights and occupancies. The applied prescriptive fire rating does not account for the reduction for sprinklers. a Conditional probability of failure of an interior column in the building in case of fire $p_{F|H_{fi}}$; b probability of occurrence of a fire in the building $p_{H_{fi}}$; c total (annual) probability of failure of an interior column in the building due to fire $p_{F,total}$.

gle column failure in taller buildings compared with low-rise buildings, as shown in Fig. 10. In order to have the same reliability level in a 12-story building as in a 3-story building, two approaches can be adopted: decrease the annual likelihood

Table 6
Application of the Method of Fig. 1 to the Case of the Prototype Buildings to Yield, from the Fragility Functions, the Annual Probabilities of Column Failure Due to Fire

No story	Fire rating (h)	Occupancy	FF$_B$ (i.e. $P_{F\|H_{fi}}(q)$)		$f_q(q)$ (MJ/m^2)		$p_{F\|H_{fi}}$ (Eq. 12)	$p_{H_{fi}}$ (Eq. 11) $\times 10^{-7}$/year	$p_{F,total}$ (Eq. 13) $\times 10^{-7}$/year
			c	ζ	μ	σ			
3	0	Office	187	0.411	420	126	0.808	2.4	1.90
3	0	Dwelling	187	0.411	780	234	0.913	5.1	4.65
3	0	Library	187	0.411	1500	450	0.897	2.4	2.11
6	1	Office	642	0.390	420	126	0.120	4.7	0.56
6	1	Dwelling	642	0.390	780	234	0.506	10.2	5.15
6	1	Library	642	0.390	1500	450	0.777	4.7	3.65
9	2	Office	988	0.386	420	126	0.017	7.1	0.12
9	2	Dwelling	988	0.386	780	234	0.213	15.3	3.25
9	2	Library	988	0.386	1500	450	0.602	7.1	4.24
12	2	Office	1000	0.367	420	126	0.013	9.4	0.12
12	2	Dwelling	1000	0.367	780	234	0.200	20.4	4.07
12	2	Library	1000	0.367	1500	450	0.600	9.4	5.65

The values are given for different building heights and occupancies

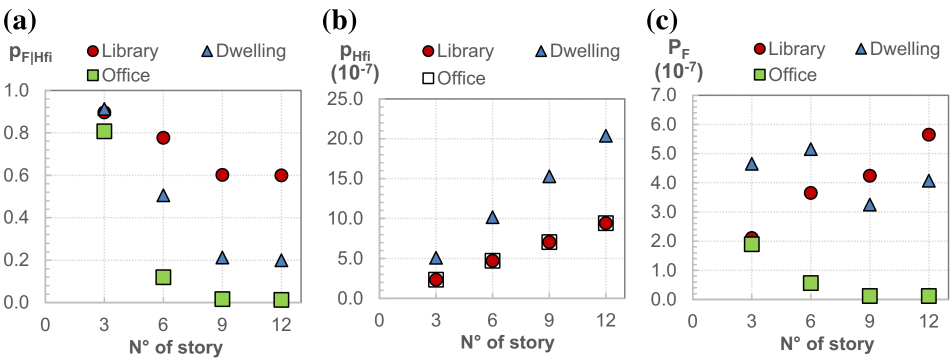

Figure 13. Comparison of fire reliability levels for sprinklered MSFB with different heights and occupancies. The applied prescriptive fire rating accounts for the reduction for sprinklers. (a) Conditional probability of failure of an interior column in the building in case of fire $p_{F|H_{fi}}$; (b) probability of occurrence of a fire in the building $p_{H_{fi}}$; (c) total (annual) probability of failure of an interior column in the building due to fire $p_{F,total}$.

of having a fire $p_{H_{fi}}$ or increase the reliability index of the elements in the taller building. The first approach is adopted when more demanding regulations are applied to taller buildings in terms of fire detection or sprinklers. The second approach requires the adjustment of the individual elements reliability targets as a function of the system scale. This idea is already partly incorporated in the IBC [38] through the adjustment of the fire rating as a function of the building height and size, as discussed in Fig. 10b. Probabilistic risk assessments as performed in this paper allow quantifying the effects of such provisions and therefore informing the selection of adequate safety targets to standardize the reliability throughout the building stock.

Note that the occupancy type also influences the total probability of failure, because it affects the term $p_{1,EN}$ in Eq. 11. For instance, the term $p_{1,EN}$ is higher for dwelling (from 4 to 9 x $10^{-7}/m^2$ year) than for an office (from 2 to 4 $\times$ $10^{-7}/m^2$ year).

5.2. Accounting for the Fire Load Distribution

The previous section has shown that the probabilities of failure can be expressed as conditional (to the occurrence of a fire) or as total, using Eq. 10 to pass from one to the other. Yet, it is important to note that the probabilities discussed in the previous section are, in both cases, function of the fire load (i.e. the intensity measure of the fragility functions). These functions are plotted on the graphs of Fig. 9 (conditional) and Fig. 10 (total) as a function of fire load. One can go a step further by adopting a probability density function (PDF) for the fire load, noted $f_q(q)$, and convolving the fragility functions by this fire load PDF, according to Eq. 12. The outcome of Eq. 12 is a scalar, $p_{F|H_{fi}}$, which yields a single value for the conditional probability of failure given a fire occurs in the building, accounting for the distribution of fire load.

$$p_{F|H_{fi}} = \int_0^\infty P_{F|H_{fi}}(q) f_q(q) dq \tag{12}$$

Figure 11 represents the two probabilistic functions in the right side of Eq. 12. It shows the FF_B for a 9-story building with a 2-h fire resistance rating and, below, the probabilistic distributions of the fire load for different occupancies. The latter are based on a Gumbel type 1 distribution with the values adopted from [52]. It can be seen that, although the fragility function does not depend on the building occupancy, the fire load does. Therefore, the conditional probability of failure $p_{F|H_{fi}}$ obtained by Eq. 12 depends on the occupancy.

5.3. Total Probability of Failure

From the results of the two previous sections, one can obtain the total probability of column failure due to fire per year using Eq. 13 (a single scalar value).

$$p_{F,total} = p_{F|H_{fi}} \times p_{H_{fi}} \tag{13}$$

In this equation, $p_{H_{fi}}$ is the annual probability of occurrence of a fire (Eq. 11), whereas $p_{F|H_{fi}}$ is the probability of failure conditional to the occurrence of a fire (scalar obtained from the convolution of the FF_B and the pdf of q, see Eq. 12).

As an example, Table 5 gives the parameters of the fragility functions FF_B (c and ζ of the lognormal distribution); the fire load densities $f_q(q)$ (μ and σ of the Gumbel distribution); the resulting conditional probability of failure $p_{F|H_{fi}}$; the annual probability of occurrence of a fire $p_{H_{fi}}$; and the resulting total probability of having a column failure due to fire per year $p_{F,total}$. The parameters for calculation of $p_{H_{fi}}$ are taken from Sect. 5.1, with values of $p_{1,EN}$ equal to 3×10^{-7}/m^2 year (office and library) and 6.5×10^{-7}/m^2 year (dwelling) [52]. The value for a library is arbitrarily taken as the same as an office because no value is given in the reference document. The columns are assumed to be heated on four sides. The building fire resistance ratings are according to the prescriptive guideline without reduction for sprinklers (i.e. 2-h, except for the 12-story which has a 3-h design). The probabilities are plotted in Fig. 12. From Table 5 and Fig. 12, it is clear that the building height and occupancy influence the $p_{F,total}$. Taller buildings and occupancies with heavy fire loads have higher probabilities of failure.

Results are similarly given in Table 6 and Fig. 13 for the case where the building fire resistance ratings are according to the IBC prescriptive guideline accounting for the reduction for sprinklers. For occupancies with low fire loads (office), the lower the building the higher the probability of failure because the fragility, and thus the fire rating, dominates the response. Due to the difference in fire protection, a 3-story office building is 15 times more likely to have a column failure due to fire any given year compared with a 9- or 12-story office building. On the other hand for occupancies with high fire loads (library), the taller the building the higher the probability of failure because the fragility is close to unity for all buildings and therefore it is the probability to have a fire that dominates. For intermediate fire loads (dwelling), the reliability remains consistent between the different building heights.

In comparison, Hopkin et al. [53] investigated the probability of fire induced structural failure for an office building in the UK with four different heights (ranging from low-rise to high-rise). The uncertainties were limited to random variables in the thermal domain. The fire resistance ratings were varied between the different buildings heights using a target reliability optimization approach. Based on their analysis, the element failure probability (1 year reference period) - range from approximately 1×10^{-5} for the Class B low-rise building (equivalent to the 3-story) protected with 1-h rating to 3×10^{-6} for the Class C mid-rise building (equivalent to the 6-story) protected with 1.5-h rating. While these values of reliabilities cannot be directly compared with Table 6, due to the absence of sprinklers in the UK examples and the discrepancies in fire ratings, a similar increase of the reliability index is observed for office buildings from low- to mid-rise.

6. Conclusion

This paper has presented probabilistic modeling techniques to assess the fire performance of structures at the system level while accounting for uncertainties. The modeling techniques rely on fire fragility functions, where local functions are first derived at the compartment level and then combined to obtain the building overall fragility. Two objectives have been addressed: (1) identify the prevailing parameters in constructing fire fragility functions for steel frame buildings, and (2) clarify the incorporation of fragility functions in a comprehensive structural fire reliability framework. The proposed method for these two objectives has been applied to multi-story steel frame buildings for interior column failure damage state.

With regards to the first objective, it is concluded that uncertainties in the fire model, the heat transfer model and the structural model all play a role in the fire fragility, so probabilistic models are required for these three steps in the analysis. In determining the demand placed on the steel structure, the compartment geometry and openings, and the thickness and conductivity of fire protection, cause large variance in the output. In contrast, density and specific heat of fire protection can be treated as deterministic. In determining the capacity of the structure, the mechanical properties of steel at elevated temperatures cause large variance in the response.

Further, at the *compartment scale*, the compartment in which the fire develops does not significantly affect the fire fragility (same conditional probability of failure). This is because the utilization ratio of the structural elements is approximately equal throughout the building by design. On the other hand, at the *building scale*, the fire resistance rating has a very strong influence on the fire fragility. Consequently, it is recommended to develop different fragility functions for buildings with different fire resistance ratings (for instance in countries that have different prescriptive requirements; or to account for the possible degradation of the fire protection due to previous earthquake or blast). However, the building height and occupancy do not influence the fire fragility; this means that the same fragility functions can be used regardless of the number of stories and the occupancy type.

With regards to the second objective, it is concluded that fragility functions can be incorporated in a comprehensive structural fire reliability framework. They need to be combined with probability distributions for the intensity measure (fire load) and for the fire ignition likelihood, in order to yield a total (i.e. unconditional) probability of failure at the building scale. When doing so, the influence of the building height and occupancy on the total probability of failure is captured, because the latter parameters affect the distribution of fire load (for the occupancy) and the probability to have a fire (for both the occupancy and the height).

The results of the second objective in terms of total probability of failure at the building scale also show that the prescriptive fire design approach does not necessarily provide the same level of safety in different buildings. In contrast, the proposed probabilistic modeling techniques can be used as part of a quantitative probabilistic risk assessment to yield a consistent reliability level.

This research, as a whole, has presented a robust methodology, which can be applied to design more fire-resilient buildings that incorporate appropriate measures of uncertainties and achieve target reliability levels. Further development of this probabilistic approach should include measures of different damage states (the current example is limited to columns), spread of fire beyond one compartment, and progressive collapse measures. In addition, in the future, the goal is to derive an inventory of fragility functions for different building typologies, to be used for fire disaster assessment of a community of buildings. As probabilistic assessments of performance through number of simulations are needed for generating the fragility functions, the process for a set of functions for different building typologies is computationally expensive. Meanwhile, the application of fragility in fire engineering is recent. The results of this paper, identifying the necessary inputs for a fire fragility function of a steel building, can be used to optimize the computational resources required for the analysis.

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